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EFFECT OF SUPERPOSITION OF STRESS RAISERS ON AXIAL-LOAD FATIGUE STRENGTH

ROGER L. COFFEY

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EFFECT OF SUPERPOSITION OF STRESS RAISERS ON AXIAL-LOAD FATIGUE STRENGTH

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Roger L. Coffey

EFFECT OF SUPERPOSITION OF STRESS RAISERS ON AXIAL-LOAD FATIGUE STRENGTH

by

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Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
MECHANICAL ENGINEERING

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ABSTRACT

A series of tests have been conducted at the U. S. Naval Postgraduate School, Monterey, California, to evaluate the effects on fatigue strength caused by the superposition of two stress-raisers. The cases of reversed bending and torsion have been previously considered.

This paper continues the investigation for axial tension of a thin flat plate of finite width. The stress raisers consist of a transverse hole and a groove. The predicted and experimental strength reduction factors are compared and stress-cycle curves are presented.

The author wishes to express his appreciation for the advice and encouragement given him by Professor Virgil M. Faires of the U. S. Naval Postgraduate School in this investigation.

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LIST OF SYMBOLS

K_f = predicted strength reduction factor

K_f' = experimental strength reduction factor

K_t = theoretical stress concentration factor

N = number of cycles to failure

q = notch sensitivity factor

q' = effective notch sensitivity factor

S_{max} = maximum stress, ksi

S_{min} = minimum stress, ksi

S_m = mean stress $\frac{S_{max} + S_{min}}{2}$, ksi

S_v = variable stress = $\frac{S_{max} - S_{min}}{2}$, ksi

S_{no} = endurance strength for fluctuating stress varying from zero to a maximum, ksi $(S_v = S_m = S_{no}/2)$

S.C.F. = stress concentration factor

S.R.F. = strength reduction factor

1. Introduction.

It has long been recognized that when a machine component is subjected to loadings that are cyclic in nature, fatigue failure may result with no prior distortion evident. The design of these components by the principles of statics alone is insufficient and even the addition of a large "safety factor" will not always give a design that is safe and reliable. The addition of stress concentrations resulting from discontinuities such as holes, keyways, changes of cross section, etc., further complicates the analysis.

The problem of designing machine components to give a reasonable lifetime of service has been approached both analytically and experimentally. The analytic approach has resulted in the development of theoretical stress concentration factors, arrived at by the theory of elasticity, which will usually give a safe but not necessarily economical design, since it has been found experimentally by fatigue tests that the actual strength reduction is usually noticeably less than the theoretical.

The problem of design is further complicated when two stress raisers act together at a point. Figure 1 shows a crankshaft journal that failed at the juncture of an oil hole and oil hole fillet. Figure 2 shows a fatigue failure that originated in a recessed fillet which had been attacked by corrosion. Although these designs were probably satisfactory with one discontinuity present, the action of the second discontinuity increased the stress concentration, and unexpected failure resulted.

A mathematical approach to the problem was first advanced by Inglis in 1913 when he adapted the elliptical hole theory to apply to a double discontinuity consisting of a notch and a crack /1/.* Several investigations were

*Numbers enclosed thus / / refer to bibliography on page 33.

FIGURE 1. FAILURE OF CRANKSHAFT JOURNAL /18/

FIGURE 2. FAILURE IN RECESSED FILLET /18/

conducted during the next two decades, primarily to investigate the effects of surface conditions on fatigue life. In general, it was noted that the combined effects of notches and surface imperfections, resulting from either machining or a normal heat treatment, were of great significance. However, no references were made to the relative importance of the individual stress concentrations.

In 1950, Mowbray /2/ conducted experiments with compound notches comprised of grooves superposed upon fillets as transition sections in width of a steel plate. Under static tensile load the elastic tensile load was calculated from the strain at the bottom of the grooves. In every case, the experimental stress concentration factor for the combination was higher than the theoretical stress concentration factor for either the groove or the fillet alone. The ratio of the experimental stress concentration factor for the compound notch to the product of the theoretical factors for the groove and fillet alone ranged from .79 to .89.

Rotating beam fatigue tests were also made with specimens having fillets, grooves, and grooves superposed on fillets. Here Mowbray found the ratio of the fatigue strength reduction factors for the compound notch to the product of the strength reduction factors for the fillet alone and for the groove alone varied from .85 to 1.12 for the various groups of experiments.

Mowbray concluded that the effective strength reduction factors for either the individual stress raisers or the superposed condition were appreciably lower in the fatigue tests than the theoretical stress concentration factors determined analytically.

In 1960, Guhse /3/ conducted fatigue tests in reversed torsion with specimens having a hole and a rough surface superposed. He found the resultant strength reduction factor was greater than the individual factors, but

less than their product.

In 1961, Bridge /4/ conducted a similar test with reversed bending and obtained similar results.

In 1962, Paul and Fawcett /5/ made a photoelastic study of the superposition of stress raising notches. The tests were conducted for static and repeated loads. They concluded that the product of the theoretical stress concentration factors for the individual stress raisers would give the superposed theoretical stress concentration factor.

The purpose of this investigation is to determine the relationship of two stress raisers consisting of a hole and groove acting together at a point in a plate of finite width under non-reversed axial tension.

The tests were conducted by the author in the Mechanical Engineering Laboratory of the U. S. Naval Postgraduate School, Monterey, California, under the supervision of Professor Virgil M. Faires during the period from March to May 1963.

2. Testing Machine.

The Sonntag Universal Fatigue Testing Machine Model No. SF-1-U, Serial No. 482451, Fig. 3, located in the Mechanical Engineering Laboratory at the U. S. Naval Postgraduate School, was used for all tests. Without the 5:1 multiplying fixture shown installed, the machine is capable of applying a maximum vertical fluctuating force of $\pm$ 1000 lbs. superposed upon a maximum static force of 1000 lbs. This allows a maximum force in either tension or compression of 2000 lbs. With the addition of the 5:1 multiplying fixture the maximum allowable force in one direction is 10,000 lbs. The alternating force is produced by an unbalanced rotating mass (D), Fig. 4, which is supported between two bearings in a cage-like frame, the top of which forms the reciprocating platen (F). The adjustable rotating mass is driven by a synchronous motor (I) so that its speed is maintained constant at 1800 rpm. The magnitude of the force is read against scale (EE). The preload is applied by deflection of the so-called "Compensator springs" (E). These springs have a spring constant of 4.1 lbs/thousandth inch.

To apply the preload, the locking wedge (R) is retracted and platform (Z') is deflected by turning the crank (Q). The amount of the deflection is read on the dial indicator (W). When the proper preload has been applied, the locking wedge is tightened.

The machine was tuned to resonance by the addition of 6 lbs. of tuning weights (AA). When a specimen breaks, the machine is cutoff by 2 reset-type cutoff switches (M), at the same time stopping the revolution counter (DD), which is driven separately by a small synchronous motor and registers the number of stress cycles in thousands.

The 5:1 multiplying fixture was modified by removing the standard grips

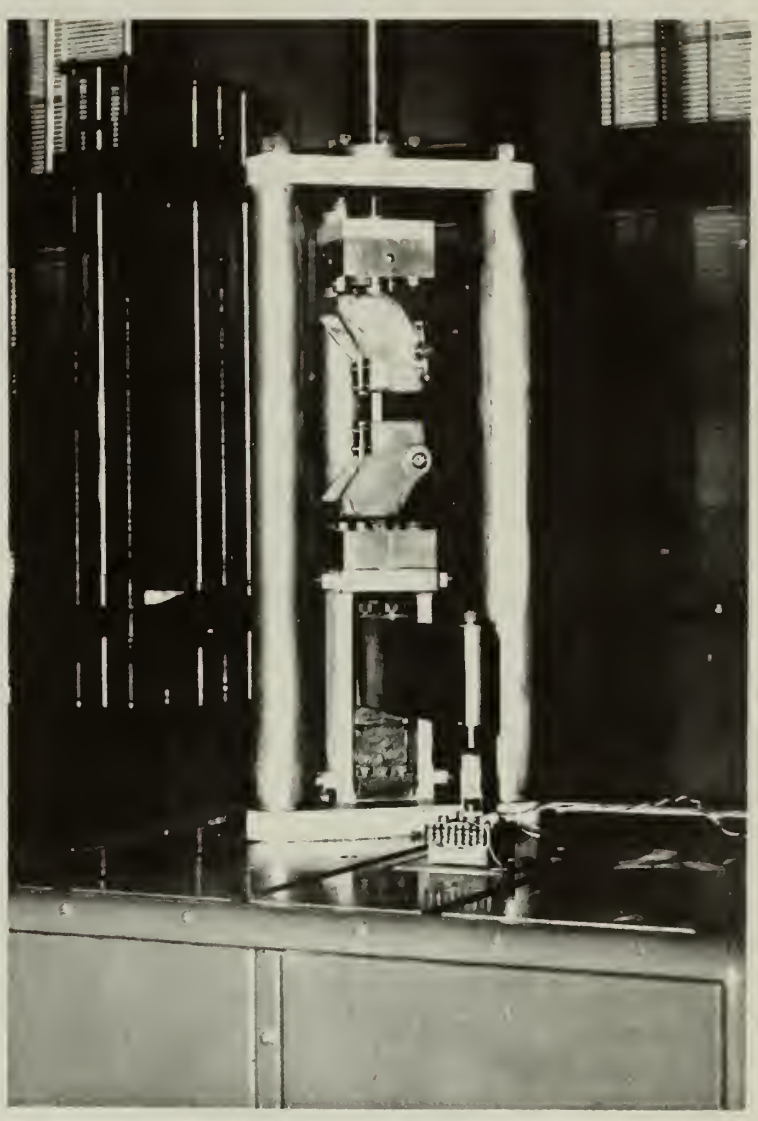


FIGURE 3.

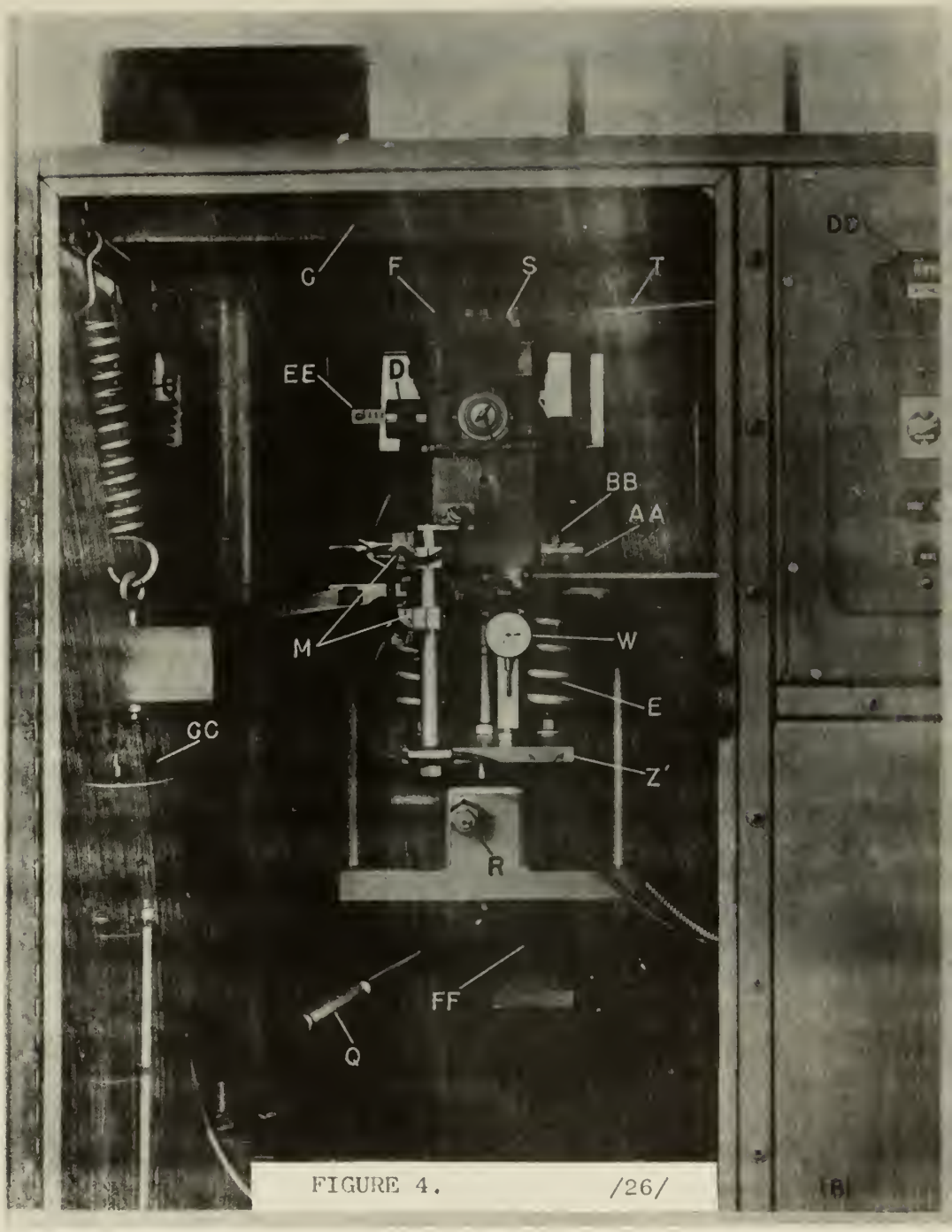


FIGURE 4. /26/

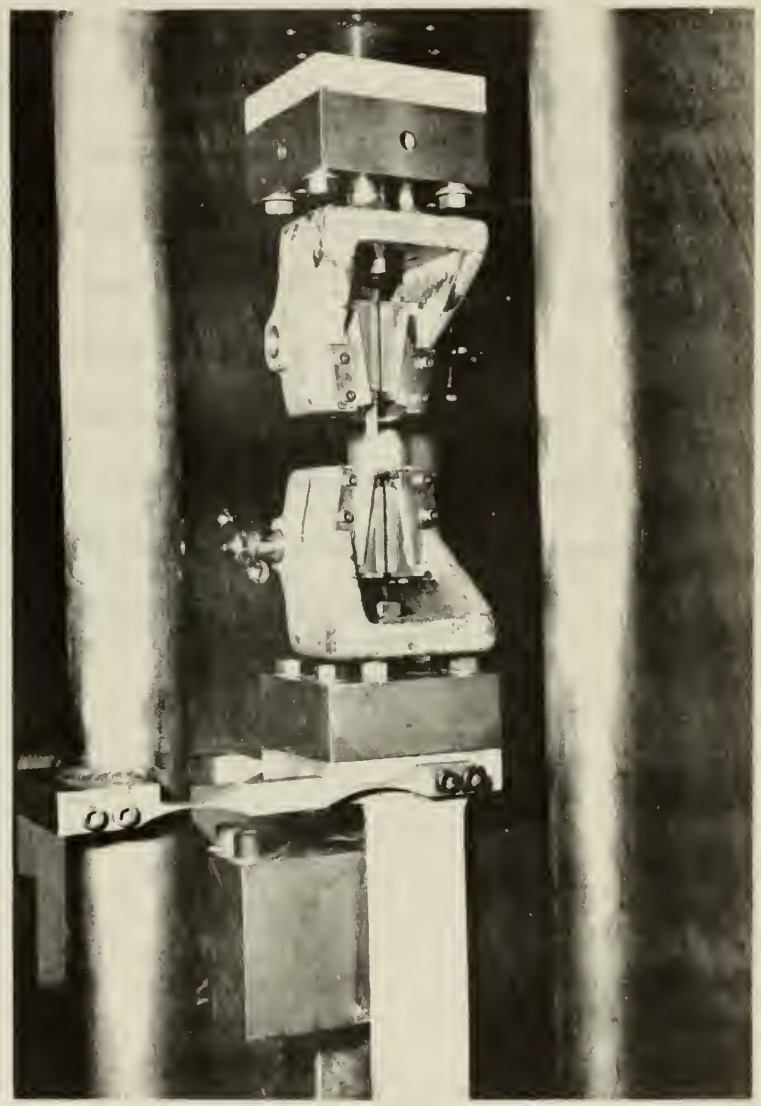


FIGURE 5.

used for round specimens and replacing them with open-face, wedge-type grips, Fig. 5, from a Riehle tensile testing machine. A four-element, strain-gage bridge was applied to the flexplate to allow monitoring of the run while the machine was in operation. The bridge was connected to an Ellis BA-2 amplifier and a Hewlett-Packard oscilloscope.

The grips were initially aligned by using a metal strip, upon which four strain gages had been mounted, in place of the specimen. Shims were added as necessary until the indicated strain due to bending, as read on a Baldwin SR-4, Type #N strain indicator, was less than 1% of the axial strain. As the investigation proceeded, slight misalignments occurred that were caused by the shock of the specimens breaking. To check the alignment, the image of an object at some distance from the machine was observed as reflected from the bright surface of the new specimen being gripped. Adjustments were made until there was no detectable motion of this image.

3. Material and Specimen Preparation..

The specimens were manufactured from a single sheet of AISI 4340 steel. This steel has a high hardenability and, for small cross sections, uniform hardness after quenching can reasonably be expected. This steel also has a high machinability rating, permitting it to be machined at relatively high hardness levels.

The certified chemical composition was: Carbon, 0.40%; Manganese, 0.70%; Phosphorous, 0.01%; Sulphur, 0.018%; Silicon, 0.28%; Chromium, 0.86%; Molybdenum, 0.28%. The sheet was received in the annealed condition and measured 1/8 x 12 x 63 inches.

The allowable length of the specimens was limited by the clearance between the grips of the machine. It was decided to use a length of 6 inches with a width in the grips of 1 inch. With allowances for cutting, this permitted 110 specimens to be made from the sheet, their major axis in the direction of rolling. The sheet was first cut into sections 6 inches long which were numbered 1 through 10. These sections were then cut into strips 1+ inch wide and numbered 1-1 to 1-11, etc. The remainder of the material was cut into coupons and used to determine the heat treatment.

The shape of the basic specimen is illustrated in Fig. 6. Since the machine has a maximum allowable load of 10,000 lbs. and since the specimens were to be heat treated to a nominal tensile strength of 200 ksi, a minimum cross sectional area of 0.05 square inches seemed reasonable. This would permit high stresses while minimizing the importance of grain size. The test-section length of 1.5 inches was expected to give a uniform stress field at the center. The 1.5-inch radius fillet permitted adequate surface for gripping with a small stress concentration factor.

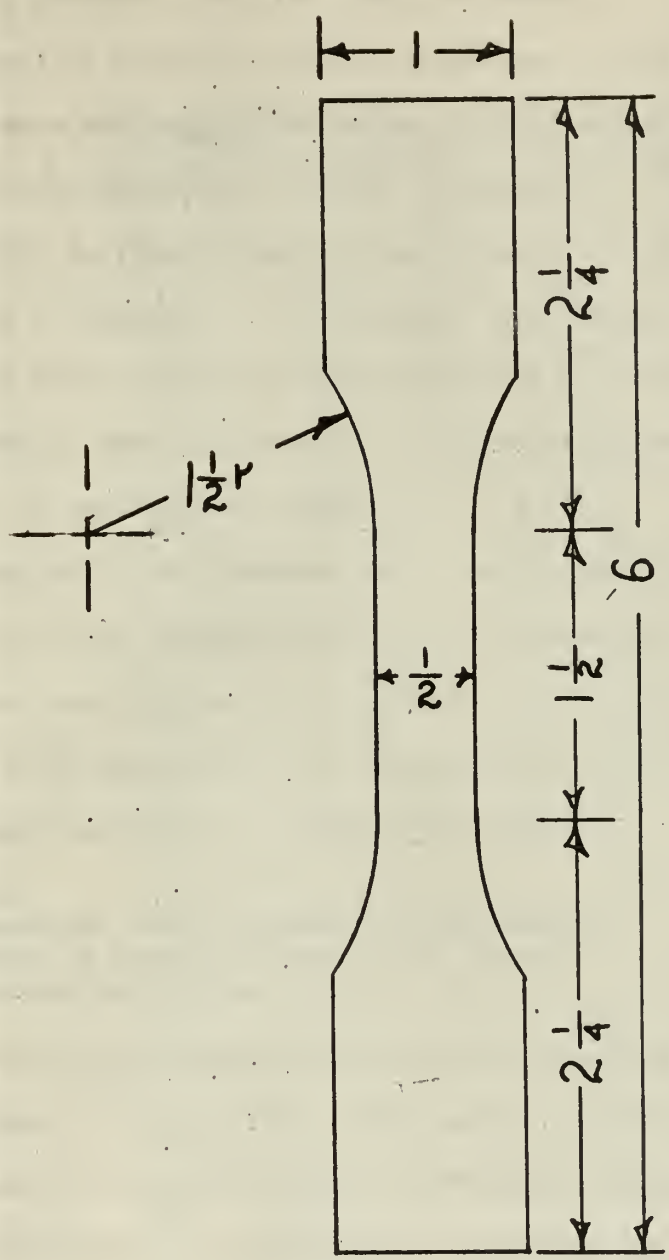


FIGURE 6 BASIC TEST SPECIMEN

The 6 x 1+ inch strips were first machined to a 1-inch width and to a thickness of 0.115 inches. A rough contour was cut in each side with a bandsaw, leaving enough material to allow removal of surface effects resulting from the heat treatment with the final machining.

The facilities of the Metallurgical Department of the U. S. Naval Postgraduate School were utilized by the author to perform the heat treatment. The specimens were initially heat treated in groups of 11 for 1-1/2 hours at 1540°F in a natural gas atmosphere and then quenched in oil, resulting in a hardness of about 63 Rockwell C. It had been found by heat treating the sample coupons in this atmosphere, and then making a microscopic inspection of the cross section, that the effects of carburization were small and would be removed in the final machining. /6, 7, 8, 9/.

The specimens were then tempered for 1 hour at 830°F and allowed to air cool. The resulting hardness was about 44.5 Rockwell C. After tempering, the specimens were divided into 4 groups, mixing them with respect to position in the sheet and position in the oven during heat treatment.

The specimens used were of 4 types. (See Fig. 7)

A. Plain
B. Transverse hole in center of test section
C. Groove on each side across test section
D. Combination of B and C.

The final machining operation consisted of contouring the test section width of all groups to 0.5 ± 0.0005 inches on the vertical mill. Groups B & D were drilled with a size 39 drill (0.0995-inch diameter), the drills being changed frequently. All groups were then ground to a thickness of 0.1 ± 0.001 inch, the last two passes being 0.00025 inch each. Groups C & D were then grooved, using a horizontal mill with a cutter having a tip radius of 0.01 inch, to a depth of 0.005 inch on each side. (This cutter is normally

FIGURE 7

used in the manufacture of Charpy impact specimens.)

After final machining, the specimens were tempered at 945° for 2 hours to stress-relieve and remove effects of machining. Four hardness readings were taken on each specimen and averaged.

A bar graph, Fig. 8, was constructed and all except five specimens fell within the $\pm 2\sigma$ limits. These five were from a group that received a double heat treatment when too much time elapsed prior to the first quench. The mean Rockwell C hardness reading for all specimens was found to be 41.32. From Fig. 9 it was estimated that the ultimate tensile strength was approximately 190,000 psi.

Next, the faces and edges of the specimens were polished by light passes over metallurgical polishing paper wrapped about a 1 1/4-inch round bar rotating in a lathe. Initial polishing with No. 1 paper was followed by polishing with 3-0 paper, giving a final surface finish estimated as 4-6 microinches when compared with a surface roughness standard. All polishing was done in a longitudinal direction since it is felt that small scratches in this direction have little effect in the initiation of fatigue cracks /10/.

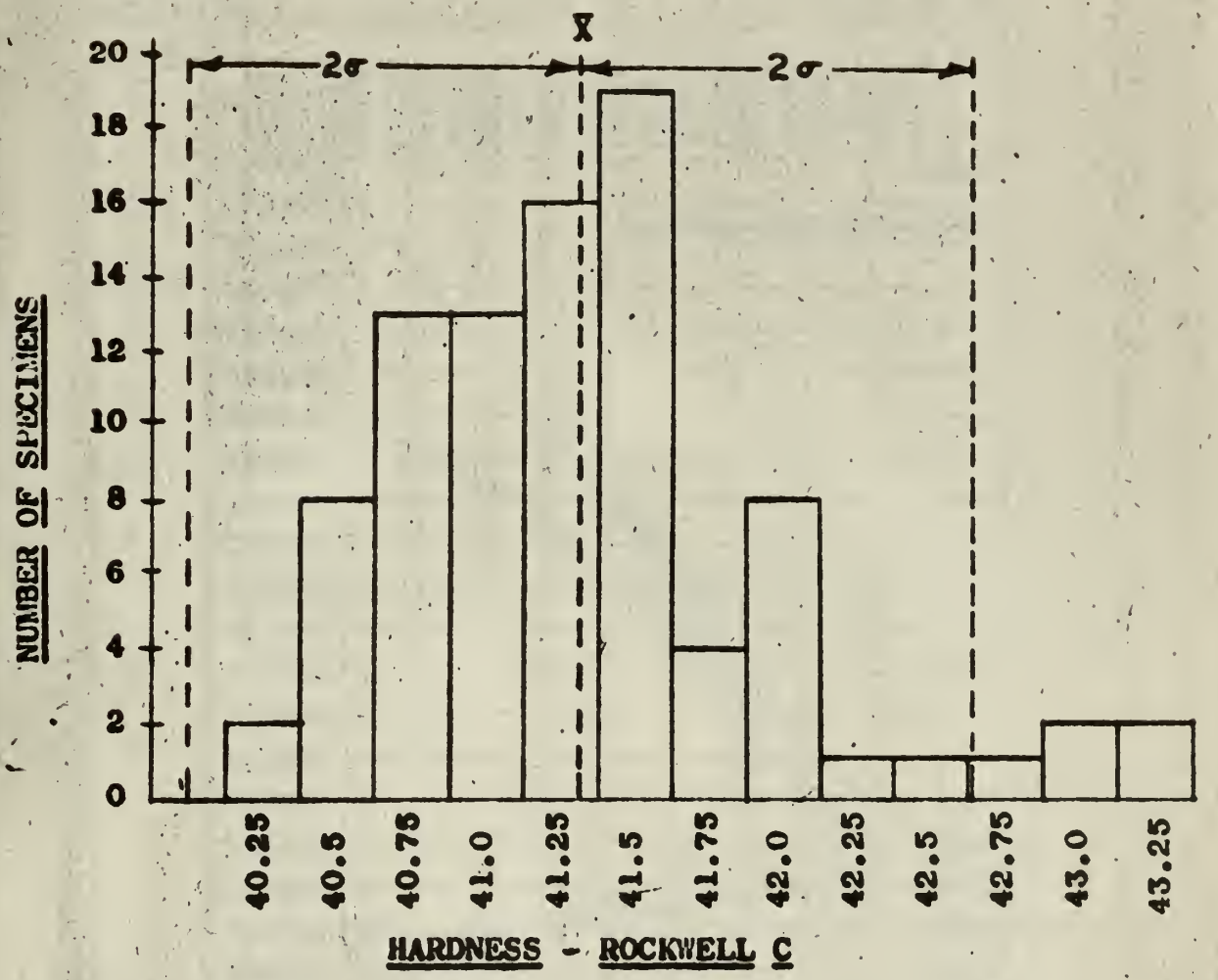


FIGURE 8 BAR GRAPH OF HARDNESS DISTRIBUTION

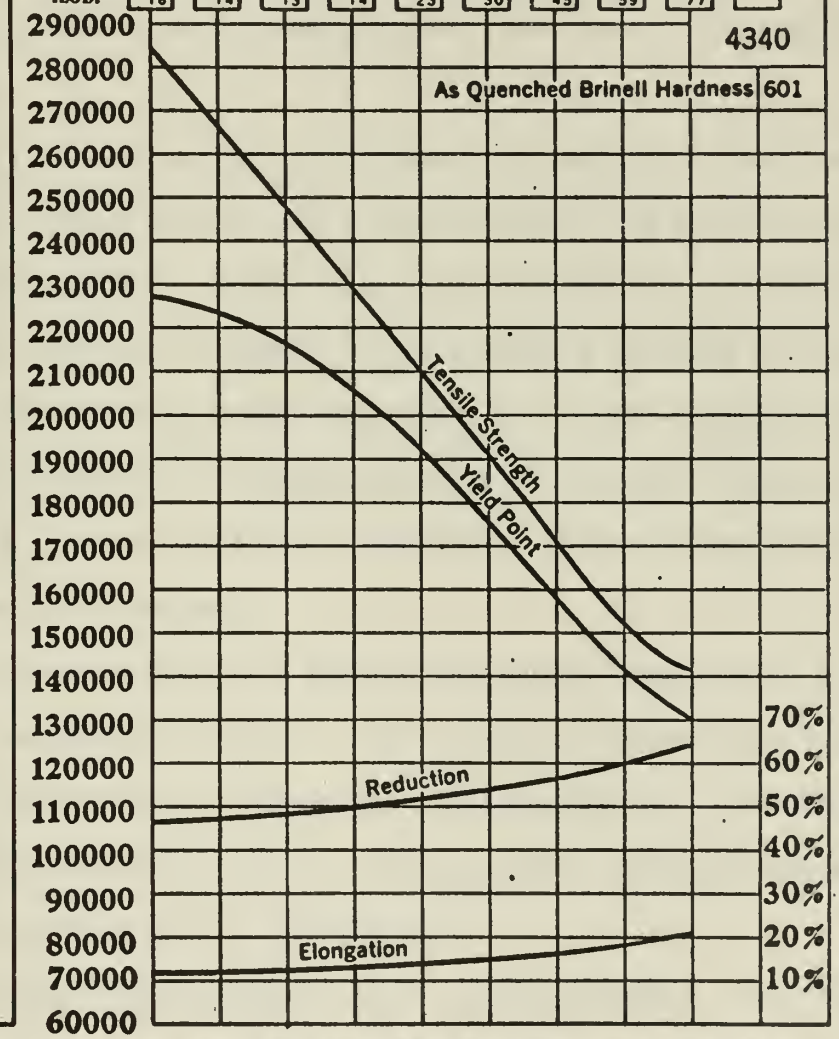
AISI-4340

(Oil Quenched)

PROPERTIES CHART

(Single Heat Results)

.530" Rd. SIZE TREATED .505" Rd. SIZE TESTED	Ac_1 1350°F. Ac_3 1415°F.	Ar_3 890°F. Ar_1 720°F.	C. .38/.43	Mn. .80/.90	P. Max. .04	S. Max. .04	Si. .20/.35	Ni. 1.65/2.00	Cr. .70/.90	Mo. .20/.30	Grain Size
HEAT TESTED			.41	.67	.023	.018	.26	1.77	.78	.26	6-8
BRIN.	555	514	477	461	415	388	363	321	293		
SHORE	73	70	66	65	59	56	52	47	43		
ROCK.	C55	C52	C50	C48	C45	C42	C39	C34	C31		
IZOD.	18	14	13	14	23	30	45	59	77		



DRAW 400°F. 500°F. 600°F. 700°F. 800°F. 900°F. 1000°F. 1100°F. 1200°F. 1300°F.

NORMALIZED AT 1600°F., REHEATED TO 1475°F. QUENCHED IN AGITATED OIL

FIGURE 9 PROPERTIES OF AISI 4340 STEEL /9/

4. Test Procedure.

The American Society for Testing Materials recommends that the "standard" test method of testing several test specimens at each of a number of different stress levels is generally the most suitable method of obtaining an S-N curve /11/. For this method, at least three stress levels must be investigated, using groups containing at least four specimens per group, if probability-stress-cycle curves are desired. In this investigation, it was planned to investigate 5 stress levels with groups of 4 specimens each. Due to difficulties encountered it was later necessary to modify this procedure.

Experiments have shown that the endurance strength for reversed axial loads is on the average approximately four-fifths of the endurance strength for reversed bending /12/. It has also been shown that as the mean stress S_m is increased there is a decrease in the endurance strength when expressed in terms of the variable stress amplitude S_v. It was proposed that this investigation establish the endurance strength in the region where $S_v = S_m = S_{no}/2$, where S_{no} is the endurance strength for a fluctuating stress varying from zero to a maximum.

Figure 10 was constructed to predict the stress levels to be first investigated with the specimens of group A. The ordinate intercept was four-fifths of the fatigue limit found in a previous investigation /4/. The Goodman line and Gerber parabola /13, 14, 15, 16/ were drawn with the ultimate strength S_u as the intercept on the abscissa.

It was thought that the stress, $S_{no}/2$, corresponding to the "knee" of the S-N curve for an axial load with zero minimum stress would be between the values A and B intercepted on a 45°-line from the origin and the Goodman and Gerber curves. From this diagram, a value of $S_{no}/2$ = 50-60 ksi was estimated.

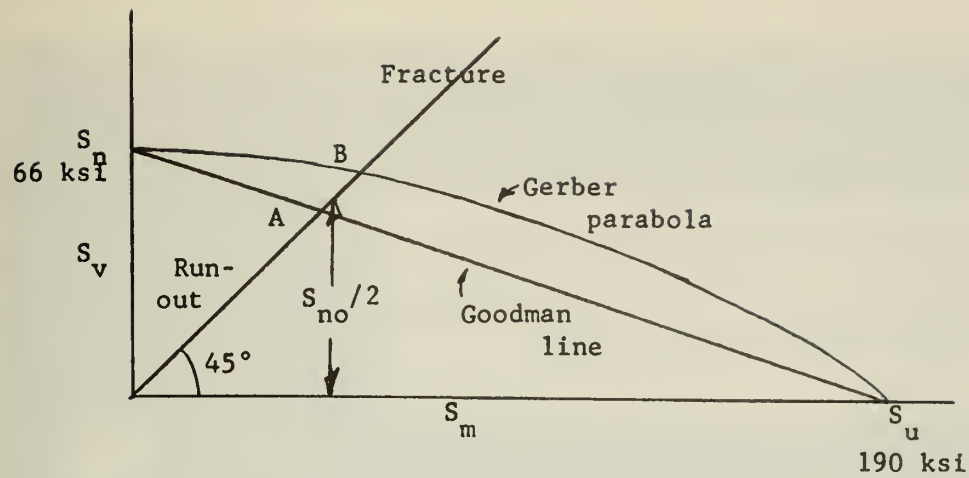


Fig. 10

Due to the wide variation in specimen dimensions, the loads required to provide a desired stress level varied with each specimen. These were computed and the specimen was preloaded to provide the desired mean stress and the unbalanced rotating mass was adjusted to provide the variable stress. Because of the wedge type grips it was necessary to maintain a tensile load on the specimen at all times during the stress cycle. Therefore the mean stress was, by necessity, always greater than the variable stress. There were 21 specimens in group A.

The specimens of group B were tested next. The diameter of the hole a was 0.0995 inches and the mean width w of the specimens in this group was 0.497 inches. Entering Fig. 11 with a/w = 0.0995/0.497 = 0.2002, a theoretical stress concentration factor $K_t = 2.515$ is obtained. To obtain the predicted strength reduction factor K_f allowance must be made for the notch sensitivity of the material, q. For quenched and tempered steels with a notch radius of $\frac{0.0995}{2} = 0.0498$ inches, Peterson /17/ suggests q = 0.95. Using the formula:

$$K_f = 1 + q\,(K_t - 1),$$

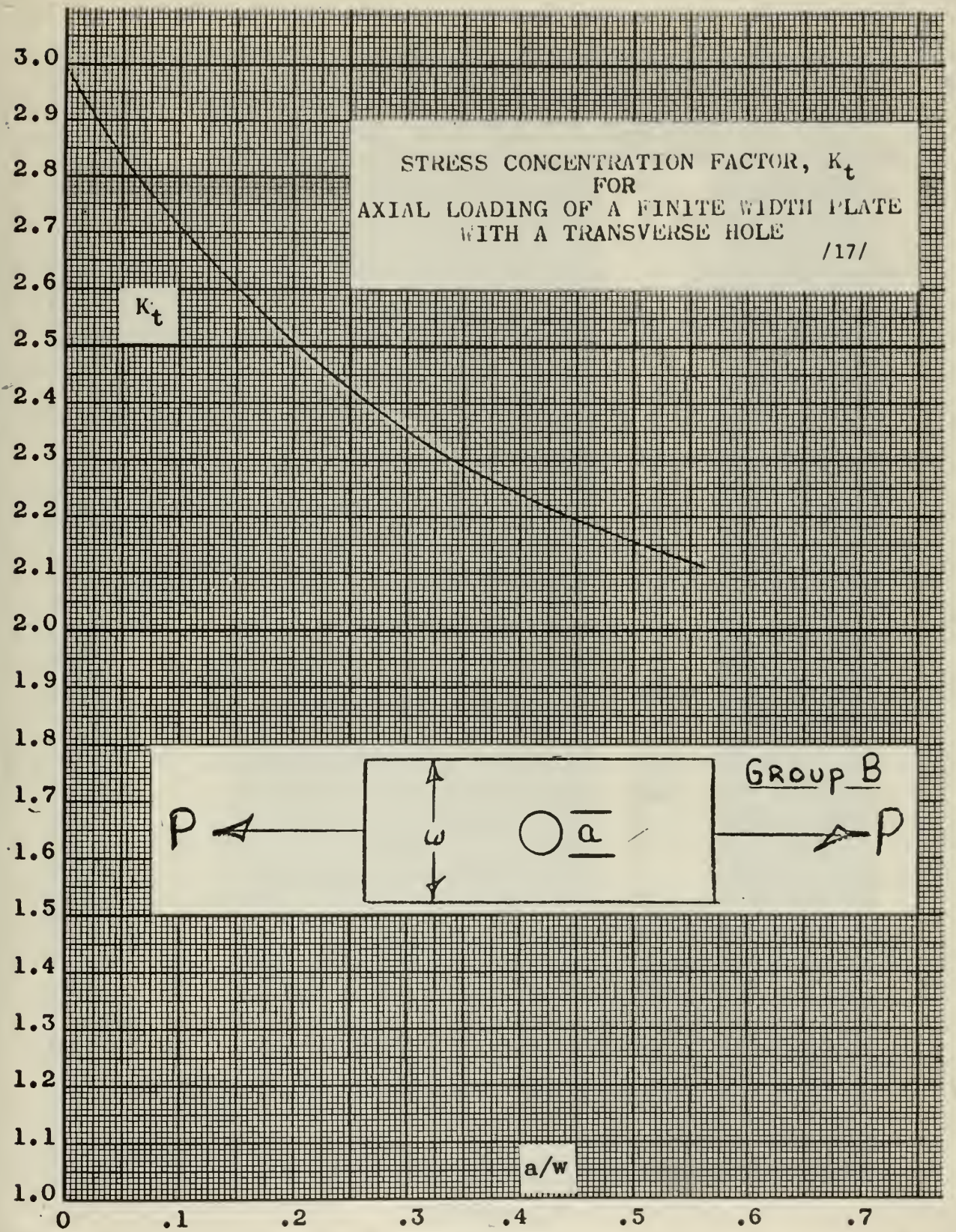

STRESS CONCENTRATION FACTOR, K_t
FOR
AXIAL LOADING OF A FINITE WIDTH PLATE
WITH A TRANSVERSE HOLE
/17/
K_t
3.0
2.9
2.8
2.7
2.6
2.5
2.4
2.3
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0
.1
.2
.3
.4
.5
.6
.7
a/w
GROUP B
P
P
w
a

FIGURE 11

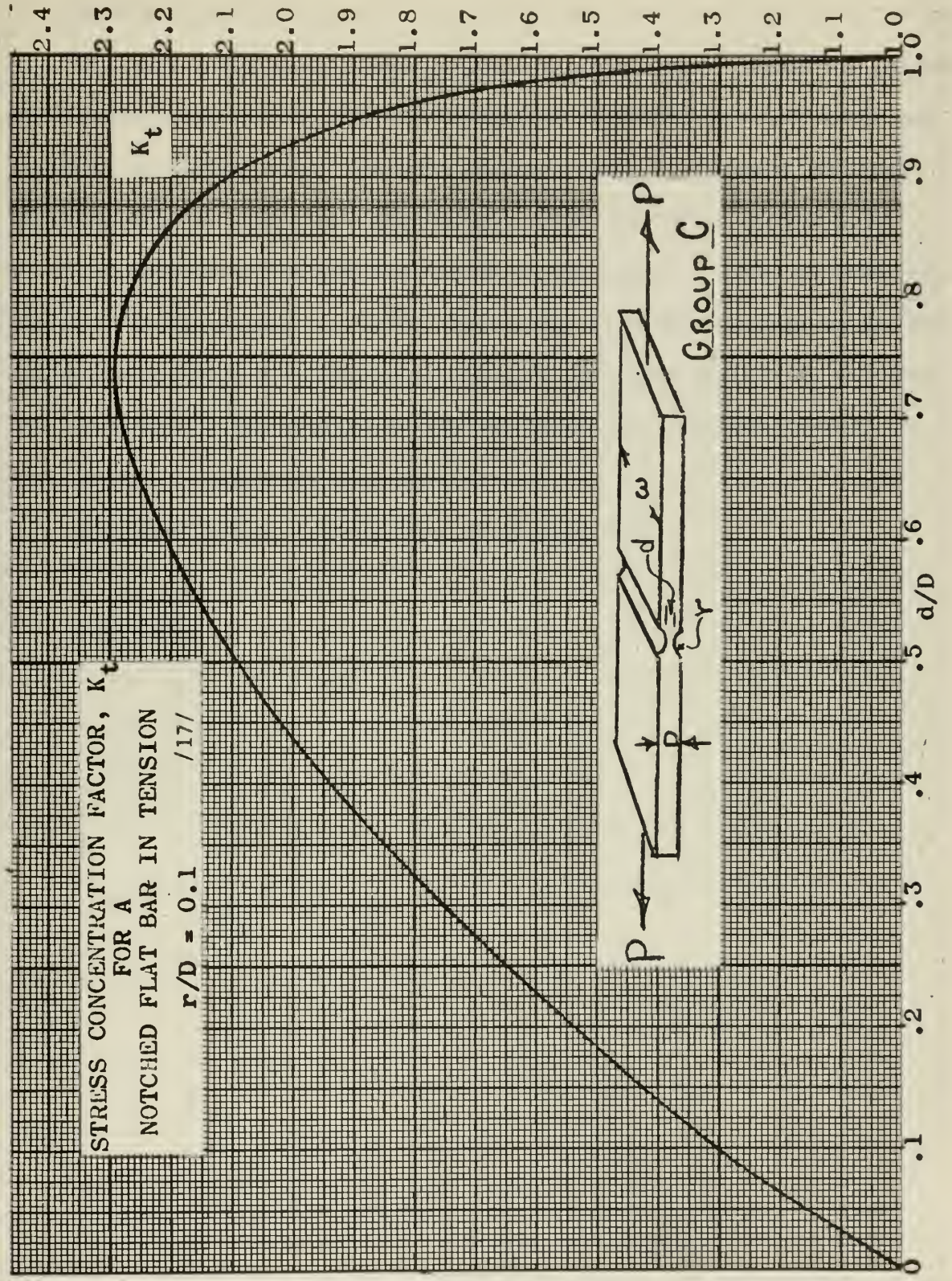

STRESS CONCENTRATION FACTOR, K_t
FOR A
NOTCHED FLAT BAR IN TENSION
r/D = 0.1 /17/
K_t
2.4
2.3
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
d/D
P
P
d
w
r
GROUP C

FIGURE 12

the predicted strength reduction was determined to be 2.439. There were 19 specimens in group B.

The groove for group C had a radius r = 0.01 inch. Since the thickness of the specimens varied, the mean effective thickness was calculated and found to be 0.0916 inch. The mean specimen thickness D was 0.101 inch. Entering Fig. 12 with r/D = 0.01/0.101 = 0.099 and d/D = 0.0916/0.101 = 0.907, a theoretical stress concentration factor, K_t = 2.09, is obtained. For this notch radius, q = 0.72, Following the same procedure as for the specimens in group B, a predicted strength reduction factor, K_f = 1.785, was calculated. The applied stress was based upon the area (w)(d). In this group, 20 specimens were tested.

The final group, D, consisted of 17 specimens.

5. Results and Discussion.

Wahl /20/ presented a modification of the usual methods of evaluating the effects of variable stresses, as applied to helical springs, which is closer to the actual physical properties for loads in which the mean stress is greater than the variable stress /21/. The basis for this design is the endurance strength under stress from zero to a maximum S_{no}.

Since it was not possible in this investigation to have an exact equality between S_m and S_v, it was necessary to perform an extrapolation to determine the equivalent value of $S_{no}/2$. An investigation of the small amount of information available for this material in this stress range indicates that the application of Wahl's theory is feasible /22/. The equation used in this extrapolation is developed in Appendix B.

Table A is a tabulation of the results obtained in this investigation. The predicted strength reduction factors, K_f, are those obtained from the literature /17/. For the superposed case, group D, the predicted factor is taken as the product of the individual factors for groups B and C. The endurance strengths S_{no} are twice the value of the variable stress amplitude $S_{no}/2$ from Fig. 13 at the endurance limit. The experimental S.R.F., K_f', is calculated from these values in accordance with the definition,

$$K_f' = \frac{\text{Stress amplitude, } S_{no}/2\text{, unnotched specimen}}{\text{Stress amplitude, } S_{no}/2\text{, notched specimen}} .$$

The major problems encountered in the investigation, summarized below, resulted from the method chosen to grip the specimens:

1. <u>Rapid wear of the grip face</u>.

   Although the grip faces were much harder than the specimen, fretting took place within the grips and the resulting wear

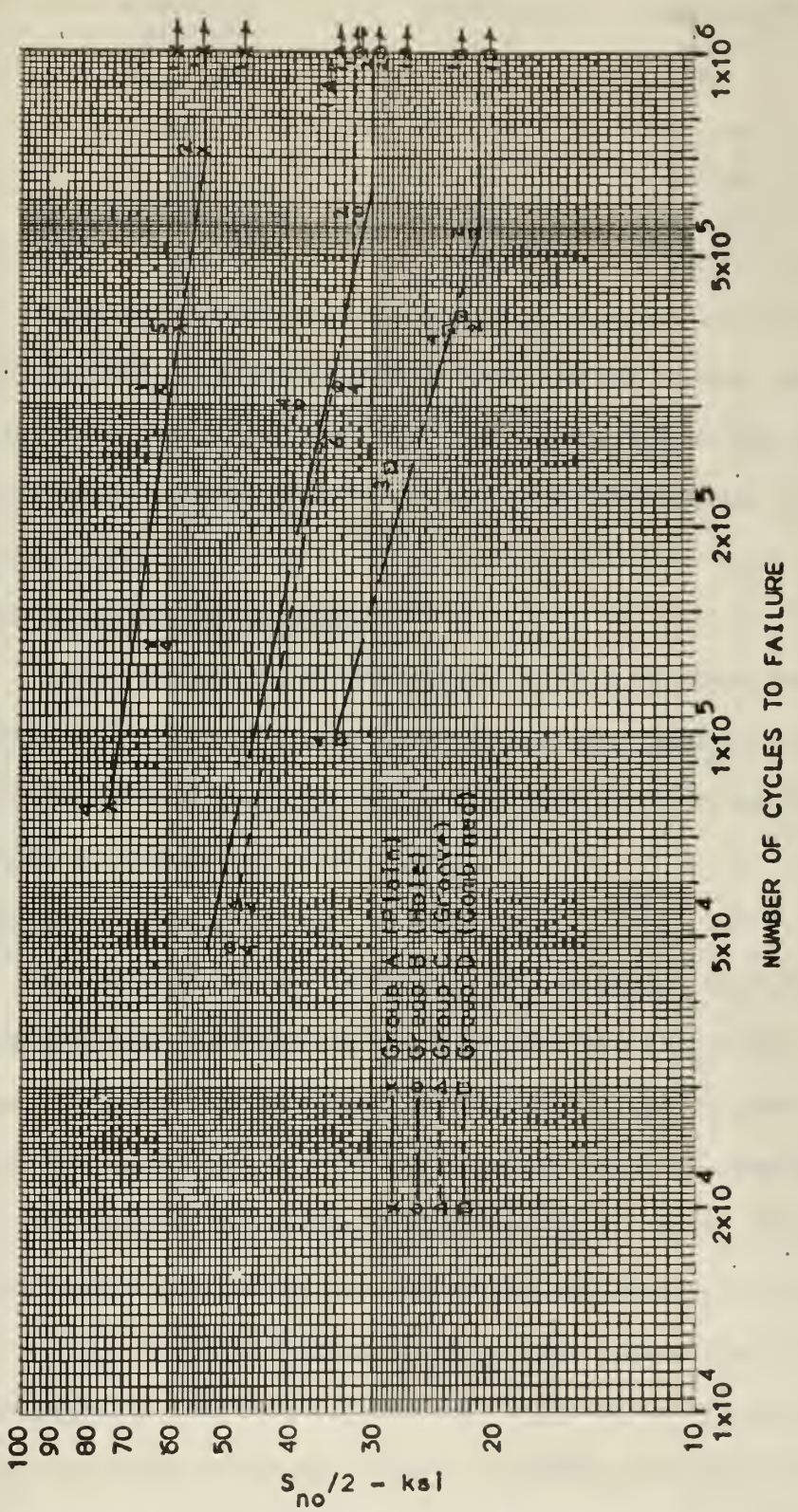


FIGURE 13 STRESS-CYCLE CURVES

TABLE A

SPECIMEN GROUP	ENDURANCE STRENGTH (S_{no}) ksi	PREDICTED S.R.F. K_f	EXPERIMENTAL S.R.F. K_f'
A	106	----	----
B	60	2.44	60/106 = 1.77
C	64	1.78	64/106 = 1.66
D	42	4.34	42/106 = 2.52

limited their usefulness to 5-6 runs. As an economy measure, grips were manufactured in the laboratory machine shop from large files. Approximately 12 sets were used during the course of the tests.

2. Uneven gripping of specimen.

Small variations in the thickness of a specimen resulted in uneven gripping and consequently in unwanted bending in the test section. The consequent uneven wear caused the situation to become progressively worse.

3. Slipping in the grips.

As the preload was applied to the specimen, some slipping took place as the teeth of the grip face set themselves. In order to allow for the variable load while running, the static load was initially set to a value greater than the maximum load and then backed off to the desired pre-load. Until the grip faces became worn, this was sufficient to prevent slipping during a run. However, during the long runs some slipping would take place as the fretting products built up in the grip face. When the slipping was sufficient to release the grip, there was uncertainty as to the value of the mean load during that portion

of the run. If the slipping was detected in time, the specimen was reloaded and the run continued.

4. Misalignment.

When a specimen breaks, the amplitude of the reciprocating platen increases until the deflection is sufficient to reach the cutoff switches. During this period the ends of the broken specimen beat together until one end is knocked from its grip. This can result in misalignment of the grips.

In view of the reasons mentioned, it was necessary to check the specimens often during a run and to observe carefully the condition of the broken ends at the completion of a run. Indications of uneven gripping and/or excessive slipping were sufficient reasons for discarding that particular run.

Of the 110 potential specimens mentioned earlier, 29 were rejected due to errors in machining. Of the 77 run, 9 were rejected for slipping, bending, or torsion, leaving 68 runs with useful results. Emphasis was placed upon determining the endurance limit for each group. Since there were fewer useful specimens per group than anticipated, the number of specimens investigated per stress level varied with their availability.

When several good runs were obtained at the same stress level the methods of statistics were used to obtain a median value for the number of cycles to failure /11/. By this method, when the observed values are arranged in order of magnitude, the median is the middlemost value and with an even number of specimens the median is the average of the two middlemost values. The curves of figure 13 were obtained by plotting these values, tabulated in Appendix A, to a larger scale and then drawing the best

line through them, weighing each point with consideration of the number of specimens run at that stress level. To obtain the endurance strength, additional consideration was given to the position of the last failure. It was felt that this point should lie above the knee of the S-N curve.

The lines drawn fall within the experimental accuracy of this investigation, since a one ksi error in their position amounts to an error of less than two per cent in the experimental S.R.F.

The endurance limit obtained for the plain specimens, group A, fell within the range of $S_{no}/2$ = 50-60 ksi predicted from figure 10. All except 4 specimens in this group failed near the fillet. The area at the point of failure was the same as that of the test section, so although the calculated stress concentration was low, it was evidently significant.

The experimental S.R.F. for the specimens of group B was much lower than the predicted S.R.F. obtained from figure 11. Since the holes were not polished, but run in the "as-machined" condition, one would expect the experimental S.R.F. to be higher rather than lower. An examination of the cracks in this group indicated that the nucleation of the crack started at the side of the hole and appeared uniform through the thickness of the cross-section. This indicated that there was no undesired notch at the juncture of the hole and the surface of the specimen which would have affected the theoretical S.C.F. or the value of the notch sensitivity.

An extensive search through the literature /14, 15, 16, 19, 27, 28/ reveals that the notch sensitivity q for small holes in bending is in general lower when obtained experimentally than the average values tabulated by Peterson /17/. However, none of these references considered the case of axial tension.

Grover /23/ conducted a series of axial-load fatigue tests with SAE

4130 steel as one of the materials used. The tests were run at several levels of mean stress and one of the types of specimens used contained a central circular hole with a theoretical S.C.F., K_t, of 2.0. For the hole used,the notch sensitivity factor q = 1 and therefore the predicted S.R.F. also equaled 2.0. At the stress levels most nearly approaching those of this investigation, the experimental S.R.F. was approximately 1.4, a 30% reduction.

Bruggeman /24/ conducted a series of axial fatigue tests on aluminum alloys using various ratios of hole diameter to specimen width, a/w. His tests were all conducted at zero mean stress, but for an a/w ratio similar to the one used in this investigation, he found the experimental S.R.F. K_f', to be 20-30% less than the predicted S.R.F., K_f,

As a result of these tests Bruggeman concluded that simply defined fatigue-strength reduction factors did not appear to have useful correlation with the theoretical stress concentration factor.

Figure 14 illustrates the mode of failure of the specimens of group B. The first indication of approaching failure was the appearance of the Lueders' lines /25/ at one side of the hole indicating the point of maximum stress concentration. There seemed to be no direct correlation between the appearance of these lines and the number of cycles before crack nucleation, travel, and eventual failure. When the run did proceed to failure, the crack would slowly proceed toward one side of the specimen, the Lueders' lines travelling with the tip of the crack. As the length of the crack increased, the nominal stress in the remaining material approached the yield strength. The actual separation was quite sudden and the sequence of events difficult to analyze. There was some indication of plastic deformation at the point of final

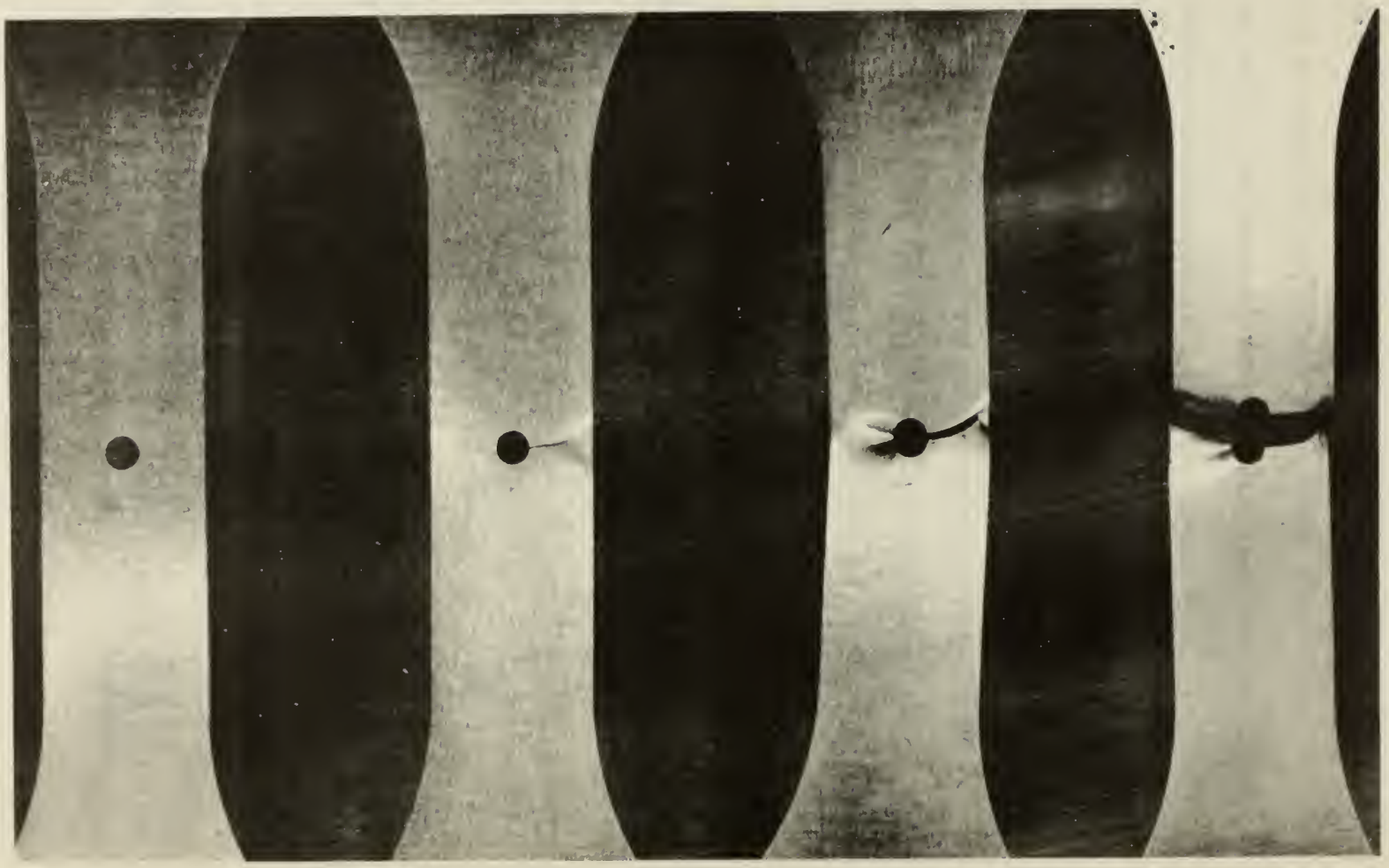


FIGURE 14. MODE OF FAILURE OF SPECIMENS WITH HOLE

failure of specimens from groups A and B. The failure of specimens from groups C and D was normally quite abrupt and no external evidence of approaching failure was visible when inspected as few as ten cycles prior to failure.

An attempt was made to obtain a correlation between the results of this investigation and those of Guhse /3/ and Bridge /4/. Since one of the stress raisers used in each of the previous investigations was surface roughness for which the effects are tabulated as a strength reduction K_f, we shall assume that the notch radius was approximately the same as that used in this investigation, r = 0.01 inch, and that the notch sensitivity would be the same, q = 0.72. With these assumptions, a reversed calculation of an equivalent theoretical S.C.F. was made. For each of the three investigations the product of the theoretical S.C.F.'s was then calculated and compared to the experimental S.R.F. obtained for the superposed stress raisers, using the formula;

$$q' = \frac{\text{experimental S.R.F.} - 1}{\text{product of theoretical S.C.F.'s} - 1}$$

where q' = the effective notch sensitivity. The results are tabulated in Table B.

TABLE B

		K_t	q	K_f	K_f'
	GUHSE				
(A)	Surface Roughness	1.46(1)	0.72(1)	1.33	1.31
(B)	Hole (r = 0.0312 in.)	1.73	1.0	1.73	1.83
	(A) (B)	2.52		2.07	1.83(2)

$$q' = \frac{1.83-1}{2.52-1} = \frac{0.83}{1.52} = 0.545$$

	BRIDGE				
(A)	Surface Roughness	1.515(1)	0.72(1)	1.37	1.52
(B)	Hole (r = 0.025 in.)	2.19	0.76	1.91	1.96
	(A) (B)	3.32		2.62	2.38(2)

$$q' = \frac{2.38-1}{3.32-1} = \frac{1.38}{2.32} = 0.595$$

TABLE B (Continued)

	COFFEY	K_t	q	K_f	K_f'
(A)	Groove	2.09	0.72	1.78	1.66
(B)	Hole (r = 0.0497 in.)	2.515	0.95	2.44	1.77
	(A) (B)	5.30		4.35	2.52(2)

$$q' = \frac{2.52-1}{5.30-1} = \frac{1.52}{4.30} = 0.354$$

(1) Equivalent values
(2) Experimental S.R.F. for superposed stress-raisers.

Although there appeared to be no direct relationship between q' and the notch sensitivity for any of the individual stress-raisers, a plot of q' versus the radius of the major stress-raisers, the hole, gave a linear relationship for this group of experiments, figure 15.

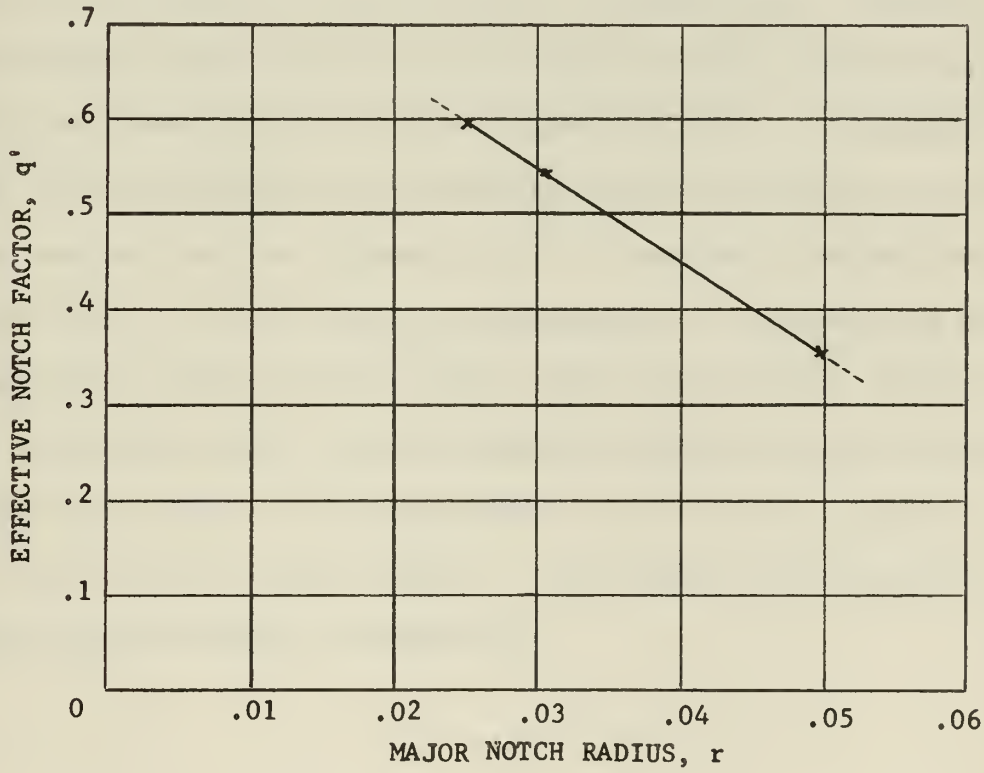


FIGURE 15

6. Conclusions

Referring to Table A, it is noted that while the experimental S.R.F. for the specimens with the groove, group C, varied from the predicted S.R.F. by only 7% of the experimental value,

$$\frac{K_f - K_f'}{K_f'} = \frac{1.78 - 1.66}{1.78} = \frac{0.12}{1.78} = 0.07$$

the experimental S.R.F. for the specimens with the transverse hole, group B, differed from the predicted S.R.F. by 40% of the experimental value. In addition, the fact that the majority of the plain specimens of group A broke near the fillet indicates the effect of either a larger stress concentration in the fillet than predicted, or the presence of undetected bending. In either case, the endurance limit of 106 ksi is possibly low. However, no feasible increase would account for the large discrepancy between the predicted and experimental S.R.F.'s for the specimens with the transverse hole. This wide variation indicates the probability that the average notch sensitivity value obtained from the literature /17/ is inaccurate for the case of the transverse hole. It is felt that an investigation to determine the true notch sensitivity factor for holes under axial loads would be of value.

The experimental S.R.F. for the superposed stress raisers of group D was greater than either of the experimental S.R.F.'s for the individual stress raisers, but only 87% of their product. This compares satisfactorily with the values found previously by Mowbray /2/.

It is interesting to note that, for this case, the product of the theoretical S.C.F.'s (2.44 x 1.78 = 4.34) is 1.7 times the experimental S.R.F. and indicates an extreme conservatism in the conclusions of Paul and Faucett /5/ when no correction is made for the notch sensitivity of the material.

The linear relationship of the three points plotted in figure 15 indicate a possibility of obtaining a family of curves for correlating the theoretical stress concentration factor to the experimental strength reduction factor in terms of an effective notch sensitivity and the major notch radius, the minor notch radius remaining fixed for each curve.

It is recommended, if further investigations are to be made into fatigue characteristics under axial loads, that the design of the specimen provide for positive holding action in the grips. Furthermore, if the nature of the investigation permits, it is recommended that round specimens be used. The specimens could be turned out much more rapidly and at the same time more satisfactory dimensional tolerances could be maintained.

BIBLIOGRAPHY

1. J. M. LESSELLS, "Strength and Resistance of Metals", John Wiley and Sons, New York, 1954, p 247.

2. A. Q. MOWBRAY, JR., "The Effect of Superposition of Stress-Raisers on Members Subjected to Static or Repeated Loads." Tech. Rep. No. 18, University of Illinois, August, 1950.

3. D. E. GUHSE, CDR, USN, "Effects of Two Stress Raisers Acting Together at a Point", M. S. Thesis, United States Naval Postgraduate School, Monterey, California, 1960.

4. J. A. BRIDGE, JR., LT, USN, "Fatigue Strength Reduction Caused by Two Discontinuities at a Point". M. S. Thesis, United States Naval Postgraduate School, Monterey, California, 1961.

5. F. W. PAUL, JR., and T. R. FAUCETT, "The Superposition of Stress Concentration Factors", Transactions of ASME, Journal of Engineering for Industry, February, 1962, pp 129-132.

6. ASM Metals Handbook, Vol. 1., "Properties and Selection of Metals", 8th Edition, 1961.

7. Gill, et al, "Tool Steels", ASM, 1946.

8. S. L. HOYT, "Metals Properties", ASME Handbook, 1954, pp 231-233.

9. Bethlehem Steel, "Modern Steels and their Properties", 6th Edition, 1961, pp 124-125.

10. B. C. HANLEY and T. J. DOLAN, "SURFACE FINISH", Metals Engineering: Design, ASME Handbook, 1953, p 101.

11. Committee E-9 on Fatigue, "A Tentative Guide for Fatigue Testing and the Statistical Analysis of Fatigue Data", ASTM STP No. 91-A, 1958.

12. V. M. FAIRES, "Design of Machine Elements", The Macmillan Co., 3rd Edition, New York, p 97.

13. ibid., p 102

14. M. HETÉNYI, "Handbook of Experimental Stress Analysis", John Wiley and Sons, New York, 1950.

15. R. CAZAUD, "Fatigue of Metals", Philosophical Library, New York, 1953, p 59.

16. J. M. LESSELLS, "Strength and Resistance of Metals", John Wiley and Sons, New York, 1954, pp 187 - 189.

17. R. E. PETERSON, "STRESS CONCENTRATION DESIGN FACTORS", John Wiley and Sons, Inc., New York, 1953.

18. T. W. BUNYAN, "Service Fatigue Failures in Marine Machinery", International Conference on Fatigue of Metals, 1956.

19. H. J. GORDON, et al, "The Fatigue of Metals and Structures", Batelle Memorial Institute, Columbus, Ohio, 1960 revision, pp 24 - 37.

20. A. M. WAHL, "Mechanical Springs", Penton Publishing Co., Cleveland, Ohio, 1944, pp 121.

21. ibid (12), p 186.

22. W. J. TRAPP and R. T. SCHWARTZ, "Elevated Temperature Fatigue Properties of SAE 4340 Steel", Proceedings ASTM, Vol. 53, 1953, p 829.

23. H. J. GROVER, et al, "Fatigue Strength of Aircraft Materials", NACA TN 2389, 1951.

24. W. C. BRUEGGEMAN and M. MAYER, JR., "Axial Fatigue Tests at Zero Mean Stress of 245-T and 755-T Aluminum-Alloy Strips with a Central Circular Hole", NACA TN 1611, 1948.

25. S. TIMOSHENKO, "Strength of Materials", Part 2, D. Van Nostrand Co., Princeton, N. J., 1956, p 330.

26. Instructional Manual, Sonntag Universal Fatigue Testing Machine, Model SF-1-U, Serial No. 482451.

27. W. J. HARRIS, "Metallic Fatigue", International Series of Monographs in Aeronautics and Astronautics, Vol. 1, Pergamon Press, New York, 1961.

28. J. A. POPE, "Metal Fatigue", Chapman and Hall Ltd., London, 1959.

APPENDIX A

TABULATED DATA

	Subject	Page
I -	Tabulation of Experimental Data	
	A. Plain specimens	36
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	C. Specimens with grooves	38
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1-A

SPECIMEN NO.	AREA inches	S_v ksi	S_{min} ksi	$S_{no}/2$ ksi	N cycles	Remarks
9-4	.0489	72.5	2.5	73.5	4.5×10^4	
10-9	.0490	72.5	2.5	73.5	6.7×10^4	
6-4	.0496	72.5	2.5	73.5	8.8×10^4	TEST SECTION
8-6	.0498	72.5	2.5	73.5	1.06×10^5	TEST SECTION
5-10	.0499	62.5	2.5	63.3	6.3×10^4	BENDING
10-11	.0496	62.5	2.5	63.3	7.9×10^4	BENDING
7-4	.0499	62.5	2.5	63.3	8.5×10^4	
10-1	.0512	62.5	2.5	63.3	1.12×10^5	
7-9	.0500	62.5	2.5	63.3	1.57×10^5	TEST SECTION
8-2	.0502	62.5	2.5	63.3	2.57×10^5	
3-3	.0493	60.9	1.5	61.4	3.13×10^5	
3-7	.0504	57.5	2.5	58.3	8.9×10^4	
5-1	.0504	57.5	2.5	58.3	1.08×10^5	
4-7	.0499	57.5	2.5	58.3	3.89×10^5	TEST SECTION
9	.0499	57.2	2.8	58.1	4.78×10^5	
3-10	.0502	57.5	2.5	58.3	1.30×10^6	
5-7	.0504	57.5	2.5	58.3	1.79×10^6	RUN OUT
7-5	.0501	52.5	2.5	53.2	2.01×10^5	
9-8	.0502	52.5	2.5	53.2	1.23×10^6	
10-8	.0501	52.5	2.5	53.2	1.50×10^6	RUN OUT
6-8	.0501	45.5	2.5	46.1	1.61×10^6	RUN OUT

1-B SPECIMEN NO.	AREA inches^2	S_v ksi	S_{min} ksi	$S_{no}/2$ ksi	N cycles	Remarks
7-2	.0388	47.7	2.5	48.3	4.0×10^4	
8-11	.0400	48.0	2.6	48.7	4.6×10^4	
5-4	.0405	47.9	2.5	48.6	5.0×10^4	
4-4	.0406	47.8	2.8	48.5	6.0×10^4	
9-9	.0406	37.65	2.5	38.2	3.8×10^4	
10-10	.0406	37.65	2.5	38.2	2.33×10^5	
5-1	.0406	37.65	2.5	38.2	3.67×10^5	
10	.0400	37.75	2.5	38.3	4.22×10^5	
6-10	.0406	33.2	1.9	33.5	1.94×10^5	+ STOPPED FOR CRACK
8-3	.04045	33.2	2.0	33.6	2.16×10^5	TORSION
6-9	.0405	33.2	2.0	33.6	2.89×10^5	
3-4	.04045	33.2	2.0	33.6	3.03×10^5	
9-1	.0405	33.2	2.0	33.6	3.32×10^5	
8-7	.0405	33.2	2.0	33.6	5.91×10^5	
7-6	.03955	31.2	1.6	31.5	5.13×10^5	
9-5	.03955	31.4	1.5	31.6	6.47×10^5	
5-8	.04035	31.2	1.4	31.4	2.12×10^6	RUN OUT
5-3	.03955	29.2	1.1	29.4	1.60×10^6	RUN OUT
6-1	.0394	29.2	1.0	29.4	1.85×10^6	RUN OUT

1-C

SPECIMEN NO.	AREA $inches^2$	S_v ksi	S_{min} ksi	$S_{no}/2$ ksi	N Cycles	Remarks
5-3	.0454	47.0	2.5	47.6	4.5×10^4	
7-3	.0452	46.7	2.8	47.4	5.5×10^4	
9-2	.0449	46.8	2.7	47.5	5.6×10^4	
3-10	.0454	46.7	2.8	47.4	6.7×10^4	
5-5	.0452	36.3	2.2	36.7	2.26×10^4	
8-4	.0455	35.8	2.7	36.3	1.33×10^5	
9-10	.0446	35.3	2.7	35.8	1.98×10^5	
6-6	.0455	35.0	2.0	35.4	3.47×10^5	
5-9	.0456	35.0	2.0	35.4	4.28×10^5	
3-1	.0454	35.0	2.0	35.4	5.00×10^5	SLIPPED
6-2	.0464	35.0	2.0	35.4	---------	SEVERE
11	.0467	34.8	2.2	35.2	---------	SLIPPING
10-6	.0459	35.0	2.0	35.4	---------	DISCONTINUED RUN
3-8	.0453	34.0	2.0	34.4	---------	SAME
5-3	.0450	34.4	2.3	34.8	8.82×10^5 +	SLIPPED
3-1	.0454	33.0	2.3	33.4	1.08×10^6	RUN OUT
9-6	.0452	3.40	-1.1	33.8	1.12×10^6 +	SLIPPED
9-2	.0449	30.8	2.2	31.2	9.56×10^5	RUN OUT
10-7	.0453	30.7	2.2	31.1	1.25×10^6 +	SLIPPED
8-8	.0454	26.4	1.1	26.6	1.57×10^6	RUN OUT

1-D

SPECIMEN NO.	AREA inches^2	S_v ksi	S_{min} ksi	$S_{no}/2$ ksi	N cycles	Remarks
7-8	.0369	33.0	2.0	33.4	7.5×10^4	
9-3	.0369	33.0	2.0	33.4	8.9×10^4	
6	.0367	33.0	2.0	33.4	1.06×10^5	
4-2	.0377	33.0	2.0	33.4	1.10×10^5	
3-9	.0369	28.0	2.0	28.3	1.76×10^5	
?	.0374	28.0	2.0	28.3	2.42×10^5	
8-5	.0369	28.0	2.0	28.3	2.64×10^5	
4-10	.0370	23.0	2.0	23.2	2.72×10^5	
8-10	.0371	23.0	2.0	23.2	3.66×10^5	
9-11	.0367	23.0	2.0	23.2	4.12×10^5	
4-6	.0362	23.0	2.0	23.2	4.23×10^5	
6-3	.0370	22.0	2.0	22.2	3.45×10^5	
5-2	.0361	22.0	2.0	22.2	4.72×10^5	
5-6	.0364	22.0	2.0	22.2	1.30×10^6	RUN OUT
8-9	.0376	21.0	2.0	21.2	3.90×10^5	
8-1	.0370	21.0	2.0	21.2	6.95×10^5	
9-3	.0369	20.0	2.0	20.2	1.20×10^6	RUN OUT

II - STRESS-CYCLE CURVE COORDINATES

	Number of Specimens	$S_{no}/2$ ksi	N Cycles
Group A			
	4	73.5	7.75×10^4
	4	63.3	1.34×10^5
	1	61.4	3.13×10^5
	5	58.3	3.89×10^5
	2	53.2	7.17×10^5
	1	46.1	
	1	53.2	$> 1.00 \times 10^6$
	1	58.3	
Group B			
	4	48.6	4.80×10^4
	4	38.2	3.00×10^5
	4	33.6	3.18×10^5
	2	31.5	5.80×10^5
	2	29.4	
	1	31.5	$> 1.00 \times 10^6$
Group C			
	4	47.5	5.55×10^4
	6	35.7	2.63×10^5
	1	34.8	8.82×10^5
	1	26.6	
	2	31.1	$> 1.00 \times 10^6$
	1	33.4	
Group D			
	4	33.4	9.75×10^4
	3	28.3	2.42×10^5
	4	23.2	3.89×10^5
	2	22.2	4.08×10^5
	2	21.2	5.40×10^5
	1	20.2	
	1	22.2	$> 1.00 \times 10^6$
	68		

APPENDIX B

SAMPLE CALCULATIONS

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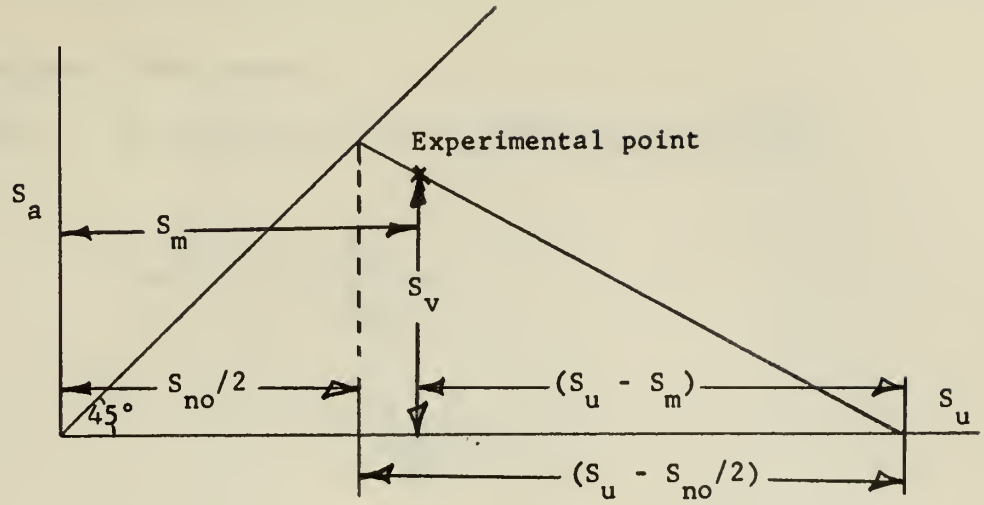
1(a).	Sample calculation of $S_{no}/2$.	43
2.	Determination of mean specimen hardness and standard deviation.	44

1. DERIVATION OF CALCULATED $S_{no}/2$.



$$\frac{S_{no}/2}{S_u - S_{no}/2} = \frac{S_v}{S_u - S_m}$$

$$\left(S_{no}/2\right) (S_u - S_m) = S_v (S_u - S_{no}/2)$$

$$\left(S_{no}/2\right) \left(1 + \frac{S_v}{S_u - S_m}\right) = \frac{S_v S_u}{S_u - S_m}$$

$$\left(S_{no}/2\right) \left[\frac{S_u - (S_m - S_v)}{S_u - S_m}\right] = \frac{S_v S_u}{S_u - S_m}$$

since, $S_m - S_v = S_{min}$

$$\boxed{\begin{aligned} S_{no}/2 &= \frac{S_v S_u}{S_u - S_{min}} \\ &= \frac{S_v}{1 - \dfrac{S_{min}}{S_u}} \end{aligned}}$$

1(a). SAMPLE CALCULATION OF $S_{no}/2$

Specimen 10-8, Group A

Data: $S_u = 190$ ksi, $S_v = 52.5$ ksi, $S_{min} = 2.5$ ksi

$$S_{no}/2 = \frac{S_v}{1 - \frac{S_{min}}{S_u}}$$

$$= \frac{52.5}{1 - \frac{2.5}{190}}$$

$$= \frac{52.5}{.9868}$$

$$= \underline{53.2 \text{ ksi}}$$

$$S_{no} = \underline{106.4 \text{ ksi}}$$

2. DETERMINATION OF MEAN SPECIMEN HARDNESS AND STANDARD DEVIATION

X	n	nX	X^2	nX^2
40.25	2	80.5	1620.0625	3240.1250
40.5	8	324.0	1640.2500	13122.0000
40.75	13	529.75	1660.5625	21587.3125
41.0	13	533.0	1681.0000	21853.0000
41.25	16	666.0	1701.5625	27225.0000
41.5	19	788.5	1722.2500	32722.7500
41.75	4	167.0	1743.0625	6972.2500
42.0	8	336.0	1764.0000	14112.0000
42.25	1	42.25	1785.0625	1785.0625
42.5	1	42.5	1806.2500	1806.2500
42.75	1	42.75	1827.5625	1827.5625
43.0	2	86.0	1849.0000	3698.0000
43.25	2	86.5	1870.5625	3741.2500
	90	3718.75		153692.5625

where,

X = Hardness at cell midpoint, R_c

n = number of specimens in each cell

Mean specimen hardness, $\bar{X}$

$$\bar{X} = \frac{\sum nX}{\sum n} = \frac{3718.75}{90} = 41.32$$

Standard deviation, σ

$$\sigma = \sqrt{\frac{\sum nX^2}{n} - \bar{X}^2}$$

$$= \sqrt{\frac{153692.5625}{90} - 1707.2598}$$

$$= \sqrt{.4353}$$

$$= .66$$

$$2\sigma = 1.32$$

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Effect of superposition of stress raiser
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